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Group action on intuitionistic fuzzy ideals of near rings

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Abstract. This article investigates the effect of group action on the intuitionistic fuzzy ideals of a near ring $\mathcal{N}$. We establish a connection between intuitionistic fuzzy $\mathcal{G}$-prime ideals and intuitionistic fuzzy prime ideals of $\mathcal{N}$. Specifically, we show that the largest $\mathcal{G}$-invariant intuitionistic fuzzy ideal contained in an intuitionistic fuzzy prime ideal, is an intuitionistic fuzzy $\mathcal{G}$-prime ideal of $\mathcal{N}$. Additionally, we demonstrate that for every intuitionistic fuzzy $\mathcal{G}$-prime ideal Q of $\mathcal{N}$, there exists an intuitionistic fuzzy prime ideal P of $\mathcal{N}$ such that $P^{\mathcal{G}} = Q$ where P is unique up to $\mathcal{G}$-orbits. The article also explores the relationship between intuitionistic fuzzy $\mathcal{G}$-prime ideals and their level cuts under group action. Moreover, we provide a characterization of intuitionistic fuzzy $\mathcal{G}$-prime ideals of $\mathcal{N}$ in terms of intuitionistic fuzzy points of $\mathcal{N}$ under this group action. Lastly, we examine the behavior of intuitionistic fuzzy $\mathcal{G}$-prime ideals of a near ring under $\mathcal{G}$-homomorphisms.

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1. Introduction

The study of group actions is a fundamental area in abstract algebra that has far-reaching implications across mathematics and its applications. At its core, a group action provides a way of understanding how the elements of a group can act on a set, preserving the structure of that set while reflecting the inherent symmetries of the group. This interaction between groups and sets forms a powerful framework for analyzing mathematical objects and their properties, enabling the classification of structures and the identification of invariants that remain unchanged under specific transformations. The formal study of group actions reveals deep connections between algebra, geometry, combinatorics, and other mathematical domains, providing essential tools for solving complex problems. The motivation to study group actions arises from their ability to generalize and unify various concepts of symmetry in mathematics and the physical world. Groups, which abstract the concept of symmetry, are fundamental in understanding the behavior of mathematical systems under transformation. Group actions not only elucidate how these symmetries act on different structures but also highlight the role of symmetry in simplifying and structuring mathematical theories. As such, group actions serve as a bridge between abstract algebra and other areas of study, enabling mathematicians to model and analyze various phenomena. The applications of group action can be found in diverse areas of science, such as physics, chemistry, biology, computer science, game theory, and cryptography, where they have been highly successful.

Since the pioneering work of Zadeh [1] on fuzzy sets, there has been a growing interest in this field due to its wide-ranging applications in engineering and computer science (See [2]). Initially, the focus was on fuzzy set theory and fuzzy logic. However, over the past two decades, there has been increasing interest in the development of fuzzy algebra, which generalizes the well-established properties of algebraic structures. One of the prominent generalizations of fuzzy set theory is the theory of intuitionistic fuzzy sets, introduced by Atanassov [3, 4, 5]. Biswas [6] introduced the notion of an intuitionistic fuzzy subgroup of a group. The concepts of intuitionistic fuzzy subring and ideal were introduced and studied by Hur et al. [7]. The concept of group action on fuzzy ideals of a ring was defined and studied by Sharma and Sharma [8]. Subsequently, Lee and Park [9] investigated the action of a group on intuitionistic fuzzy ideals of a ring R and found a relationship between the intuitionistic fuzzy $\mathcal{G}$-prime ideals of R and the intuitionistic fuzzy ideals of R. Sharma [10], studied the group action on intuitionistic fuzzy modules. Later, Yilmaz et al. [11] defined the intuitionistic fuzzy action of a group on a set. Ali and Smarandache [12] studied group action through the application of fuzzy sets and neutrosophic sets, while Manemaran and Nagaraja [13] applied group action on picture fuzzy soft $\mathcal{G}$-modules.

In this paper, we define the group action on a near ring $\mathcal{N}$ and, using this, explore the study of group action on intuitionistic fuzzy ideals of $\mathcal{N}$, $\mathcal{G}$-invariant intuitionistic fuzzy ideals, and $\mathcal{G}$-primeness of intuitionistic fuzzy ideals of $\mathcal{N}$. We also investigate a connection between $\mathcal{G}$-invariant intuitionistic fuzzy ideal, intuitionistic fuzzy prime ideal and the intuitionistic fuzzy $\mathcal{G}$-prime ideal of $\mathcal{N}$. In addition to this we also study intuitionistic fuzzy $\mathcal{G}$-prime ideal of $\mathcal{N}$ in terms of intuitionistic fuzzy points of $\mathcal{N}$ under this group action. Finally, we investigate the behavior of group action on the intuitionistic fuzzy $\mathcal{G}$-ideals under $\mathcal{G}$-homomorphism.

2. Preliminaries

A set $\mathcal{N}$ with two binary operations $+$ and $\cdot$ is known as *left near ring*, if (i) $(\mathcal{N},+)$ is a group (not necessarily abelian), (ii) $(\mathcal{N},\cdot)$ is a semigroup, (iii) $x\cdot(y+z)=x\cdot y+x\cdot z\ \forall x,y,z\in\mathcal{N}$. Analogously, $\mathcal{N}$ is said to be a *right near ring*, if $\mathcal{N}$ satisfies (iii)$'$ $(x+y)\cdot z=x\cdot z+y\cdot z\ \forall x,y,z\in\mathcal{N}$. A near ring $\mathcal{N}$ with $0\cdot x=0\ \forall x\in\mathcal{N}$, is known as *zero symmetric*, if $0\cdot x=0$, (left distributivity yields that $x\cdot 0=0$). Throughout the paper, $\mathcal{N}$ represents a zero symmetric left near ring; for simplicity, we call it a *near ring*. An *ideal* of near ring $(\mathcal{N},+,\cdot)$ is a subset $\mathcal{M}$ of $\mathcal{N}$ such that (i) $(\mathcal{M},+)\trianglelefteq(\mathcal{N},+)$ (ii) $\mathcal{N}\mathcal{M}\subseteq\mathcal{M}$, (iii) $(n_1+m)n_2-n_1n_2\in\mathcal{M}\ \forall n_1,n_2\in\mathcal{N},m\in\mathcal{M}$. Note that if $\mathcal{M}$ fulfils (i) and (ii), it is referred to as a *left ideal* of $\mathcal{N}$. It is termed a *right ideal* of $\mathcal{N}$, if $\mathcal{M}$ satisfies (i) and (iii). A mapping $\phi:\mathcal{N}\to\mathcal{N}'$ from near ring $\mathcal{N}$ to near ring $\mathcal{N}'$ is said to be a *homomorphism*, if (i) $\phi(x+y)=\phi(x)+\phi(y)$ (ii) $\phi(xy)=\phi(x)\phi(y)\ \forall x,y\in\mathcal{N}$. A homomorphism $\phi:\mathcal{N}\to\mathcal{N}$ which is bijective is said to be an *automorphism* on $\mathcal{N}$. The set of all automorphism of $\mathcal{N}$ denoted by $Aut(\mathcal{N})$ forms a group under the operation of composition of mappings. The detail of results about near rings can be found in the book [14].

Definition 2.1 ([3, 4, 5])**.** An *intuitionistic fuzzy set* (IFS) A in X can be represented as an object of the form $A=\{\langle x,\mu_A(x),\nu_A(x)\rangle : x\in X\}$, where the functions $\mu_A:X\to[0,1]$ and $\nu_A:X\to[0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x\in X$ to A respectively and $0\leq\mu_A(x)+\nu_A(x)\leq 1$ for each $x\in X$.

Remark 2.2 ([5, 15, 16, 17])**.** (1) =If $\mu_A(x)+\nu_A(x)=1$ for every $x\in X$, then A is termed as a fuzzy set.

(2) An IFS $A=\{\langle x,\mu_A(x),\nu_A(x)\rangle : x\in X\}$ is briefly written as $A(x)=(\mu_A(x),\nu_A(x))$ for every $x\in X$. We designate it by $IFS(X)$ the collection of all IFSs of X.

(3) If $p,q\in[0,1]$ such that $p+q\leq 1$, then $A\in IFS(X)$ defined by $\mu_A(x)=p$ and $\nu_A(x)=q$ for every $x\in X$, is called a *constant IFS* of X. Any IFS of X defined other than this is referred to as a non-constant intuitionistic fuzzy set.

If $A, B \in IFS(X)$, then $A \subseteq B$ contingent upon $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for every $x \in X$ and $A = B \Leftrightarrow A \subseteq B$ and $B \subseteq A$. Given any subset Y of X, the IF-characteristic function χ_Y is an IFS of X, described as $\chi_Y(x) = (1, 0)$, for every $x \in Y$ and $\chi_Y(x) = (0, 1)$ for every $x \in X \backslash Y$. Assume that $\alpha, \beta \in [0, 1]$ such that $\alpha + \beta \leq 1$. Then the crisp set $A_{(\alpha,\beta)} = \{x \in X : \mu_A(x) \geq \alpha$ and $\nu_A(x) \leq \beta\}$ is termed as the (α, β)-*level cut subset* of A. Further, if $f : X \to Y$ is a mapping and A, B be respectively IFS of X and Y. The *image* $f(A)$ is an IFS of Y is described as $\mu_{f(A)}(y) = \sup\{\mu_A(x) : f(x) = y\}$, $\nu_{f(A)}(y) = \inf\{\nu_A(x) : f(x) = y\}$ for all $y \in Y$ and the *inverse image* $f^{-1}(B)$ is an IFS of X is described as $\mu_{f^{-1}(B)}(x) = \mu_B(f(x))$, $\nu_{f^{-1}(B)}(x) = \nu_B(f(x))$, for every $x \in X$, i.e., $f^{-1}(B)(x) = B(f(x))$, for every $x \in X$ (See [16]).

Definition 2.3 ([6]). Let $(\mathcal{G}, +)$ be a group and A be an IFS in $\mathcal{G}$. Then A is called an *intuitionistic fuzzy subgroup* (IFSG) of $\mathcal{G}$, if the following hold: for all $g_1, g_2, g \in \mathcal{G}$,

(i) $\mu_A(g_1 + g_2) \geq \min\{\mu_A(g_1), \mu_A(g_2)\}$, (ii) $\mu_A(-g) = \mu_A(g)$,
(iii) $\nu_A(g_1 + g_2) \leq \max\{\nu_A(g_1), \nu_A(g_2)\}$, (iv) $\nu_A(-g) = \nu_A(g)$.

An IFSG A of $\mathcal{G}$ is called *normal* (or an *intuitionistic fuzzy normal subgroup* (IFNSG) of $\mathcal{G}$), if

$$\mu_A(g_1 + g - g_2) = \mu_A(g) \text{ and } \nu_A(g_1 + g - g_2) = \nu_A(g)\ \forall g_1, g_2, g \in \mathcal{G} \text{ (See [18]).}$$

Definition 2.4 ([19, 20]). An IFS A of a near ring $\mathcal{N}$ is said to be an *intuitionistic fuzzy subnear ring* (IFSNR) of $\mathcal{N}$, if A is an IFSG of $\mathcal{N}$ with respect to the addition $+$ and is an intuitionistic fuzzy groupoid with respect to the multiplication $\cdot$, i.e.,

(i) $\mu_A(r - s) \geq \min\{\mu_A(r), \mu_A(s)\}$, (ii) $\nu_A(r - s) \leq \max\{\nu_A(r), \nu_A(s)\}$,
(iii) $\mu_A(rs) \geq \min\{\mu_A(r), \mu_A(s)\}$, (iv) $\nu_A(rs) \leq \max\{\nu_A(r), \nu_A(s)\}\ \forall r, s \in \mathcal{N}$.

Definition 2.5 ([19, 20]). Let $A \in IFS(\mathcal{N})$. Then A is termed as an *intuitionistic fuzzy ideal* (IFI) of a near ring $\mathcal{N}$, if the subsequent conditions hold: for every $r, s \in \mathcal{N}$,

(i) A is an IFSNR, (ii) A is an IFNSG of $(\mathcal{N}, +)$,
(iii) $\mu_A(rs) \geq \mu_A(s)$, (iv) $\nu_A(rs) \leq \nu_A(s)$,
(v) $\mu_A((r + t)s - rs) \geq \mu_A(t)$, (vi) $\nu_A((r + t)s - rs) \leq \nu_A(t)$.

If A satisfies (i)-(iv), then A is called an intuitionistic fuzzy left ideal of $\mathcal{N}$. If A satisfies (i), (ii), (v) and (vi), then it is called an *intuitionistic fuzzy right ideal* of $\mathcal{N}$.

Note that $\mu_A(0) \geq \mu_A(r) \geq \mu_A(1), \nu_A(0) \leq \nu_A(r) \leq \nu_A(1)\ \forall r \in \mathcal{N}$. The collection of all IFIs of $\mathcal{N}$ is abbreviated by $IFI(\mathcal{N})$.

Definition 2.6 ([15, 17, 21]). Suppose $A, B \in IFI(\mathcal{N})$. Then the intuitionistic fuzzy product $A \circ B$ of A and B is describe as: for every $x \in \mathcal{N}$,

$$(\mu_{A\circ B}(x), \nu_{A o B}(x)) = \begin{cases} (\sup_{x=rs} \min\{\mu_A(r), \mu_B(s)\}, \inf_{x=rs} \max\{\nu_A(r), \nu_B(s)\}) & \text{if } x = rs \\ (0, 1) & \text{otherwise,} \end{cases}$$

where as usual supremum and infimum of an empty set are taken to be 0, 1 respectively.

Remark 2.7 ([15, 16, 19]). Let $\mathcal{N}$ be a commutative ring. Then for any $a_{(p,q)}, b_{(t,s)} \in IFP(\mathcal{N})$

(i) $a_{(p,q)} + b_{(t,s)} = (a + b)_{(p\wedge t, q\vee s)}$; (ii) $a_{(p,q)} b_{(t,s)} = (ab)_{(p\wedge t, q\vee s)}$.

Theorem 2.8 ([19, 20]). *Let $A \in IFS(\mathcal{N})$. Then A is an intuitionistic fuzzy ideal if and only if $A_{(\alpha,\beta)}$ is an ideal of $\mathcal{N}$ for all $\alpha \leq \mu_A(0), \beta \geq \nu_A(0)$ with $\alpha + \beta \leq 1$. In particular, if A is an IFI of $\mathcal{N}$, then $A_* = \{x \in R : \mu_A(x) = \mu_A(0), \nu_A(x) = \nu_A(0)\}$ is always an ideal of $\mathcal{N}$.*

Definition 2.9 ([15, 16, 17]). Let P be a non constant IFI of a near ring $\mathcal{N}$. Then P is designate to be an *IF prime ideal* of $\mathcal{N}$, if for any two IFIs A, B of $\mathcal{N}$ with $A \circ B \subseteq P$, $A \subseteq P$ or $B \subseteq P$.

Theorem 2.10 ([15, 16, 17]). *Let P be an IFI of a near ring $\mathcal{N}$. Then given any $a_{(p,q)}, b_{(t,s)} \in IFP(\mathcal{N})$ the subsequent conditions are identical:*

(1) *P is an IF prime ideal of $\mathcal{N}$,* (2) *$a_{(p,q)} b_{(t,s)} \subseteq P \Rightarrow a_{(p,q)} \subseteq P$ or $b_{(t,s)} \subseteq P$.*

Theorem 2.11 ([16, 17]). *If P is an IF prime ideal of $\mathcal{N}$, then the subsequent conditions hold:*

(1) *$P(0) = (1, 0)$,*
(2) *P_* is a prime ideal,*
(3) *$Img(P) = \{(1, 0), (t, s)\}$ for all $t, s \in [0, 1)$ with $t + s \leq 1$.*

Definition 2.12 ([16]). Let X and Y are any sets, $f : X \to Y$ be any function. An IFS A of X is called *f-invariant*, if $f(x_1) = f(x_2) \Rightarrow \mu_A(x_1) = \mu_A(x_2)$ and $\nu_A(x_1) = \nu_A(x_2)$ $\forall x_1, x_2 \in X$.

Proposition 2.13. *Let $f : \mathcal{N} \to \mathcal{N}^{'}$ be a near ring homomorphism from $\mathcal{N}$ onto $\mathcal{N}^{'}$ and $A \in IFS(\mathcal{N})$ which is a constant on $ker f$. Then $f(A)(f(r)) = A(r)$ $\forall r \in \mathcal{N}$.*

Theorem 2.14 ([17]). *Let $\mathcal{N}$ and $\mathcal{N}^{'}$ be near rings. Let $f : \mathcal{N} \to \mathcal{N}^{'}$ be a homomorphism. If P be an IF prime ideal of $\mathcal{N}^{'}$, then $f^{-1}(P)$ is an IF prime ideal of $\mathcal{N}$.*

Theorem 2.15 ([17]). *Let $\mathcal{N}$ and $\mathcal{N}^{'}$ be near rings. Let $f : \mathcal{N} \to \mathcal{N}^{'}$ be an epimorphism. If P is an IF prime ideal which is constant on $Ker f$ of $\mathcal{N}$, then $f(P)$ is an IF prime ideal of $\mathcal{N}^{'}$.*

3. Group action on intuitionistic fuzzy ideal of a near ring

Definition 3.1 ([22]). Let $\mathcal{G}$ be a group and $\mathcal{S}$ a non-empty set. Then the map $\phi : \mathcal{G} \times \mathcal{S} \to \mathcal{S}$, with $\phi(g, x)$ written as $g * x$, is called an *action* of $\mathcal{G}$ on $\mathcal{S}$, if for all $g, h \in \mathcal{G}, x \in \mathcal{S}$,
(i) $g * (h * x) = (gh) * x$,
(ii) $e * x = x$, where e is the identity element of the group $\mathcal{G}$.

We assume that $\mathcal{N}$ is a near ring and $\mathcal{G} = Aut(\mathcal{N})$ is a group of automorphism of $\mathcal{N}$. Then $\mathcal{G}$ acts on $\mathcal{N}$ defined by $\phi(g, x) = g(x), i.e., g * x = g(x)$. Here, we define the group action of $\mathcal{G}$ on an IFS A of a near ring N.

Definition 3.2. The *group action* of $\mathcal{G}$ on an IFS A of a near ring $\mathcal{N}$, denoted by A^g, is defined as
$$A^g = \{\langle x, \mu_{A^g}(x), \nu_{A^g}(x)\rangle : x \in \mathcal{N}\},$$
where $\mu_{A^g}(x) = \mu_A(x^g)$ and $\nu_{A^g}(x) = \nu_A(x^g)$ for every $x \in \mathcal{N}, g \in \mathcal{G}$.

Example 3.3. Let $\mathcal{N} = \{0, 1, 2\}$ be a set. Then under the two binary operations addition mod(3) and multiplication mod(3) $\mathcal{N}$ from a zero symmetric near ring. The only two automorphisms are (i) identity map and (ii) the map g defined by $g(0) = 0, g(1) = 2$ and $g(2) = 1$. Define the IFS A on $\mathcal{N}$ as
$$\mu_A(x) = \begin{cases} 0.8 & \text{if } x = 0 \\ 0.6 & \text{if } x = 1 \\ 0.5 & \text{if } x = 2, \end{cases} \qquad \nu_A(x) = \begin{cases} 0.1 & \text{if } x = 0 \\ 0.2 & \text{if } x = 1 \\ 0.3 & \text{if } x = 2. \end{cases}$$
Take $g \in \mathcal{G} = Aut(N)$. Then the action of the element g on A is defined as
$\mu_{A^g}(x) = \mu_A(x^g) = \mu_A(g(x))$ and $\nu_{A^g}(x) = \nu_A(x^g) = \nu_A(g(x))$ is given by
$$\mu_{A^g}(x) = \begin{cases} 0.8 & \text{if } x = 0 \\ 0.5 & \text{if } x = 1 \\ 0.6 & \text{if } x = 2, \end{cases} \qquad \nu_{A^g}(x) = \begin{cases} 0.1 & \text{if } x = 0 \\ 0.3 & \text{if } x = 1 \\ 0.2 & \text{if } x = 2. \end{cases}$$

Example 3.4. Consider $\mathcal{N} = \mathbf{Z}_2 \times \mathbf{Z}_2 = \{(0, 0), (1, 0), (0, 1), (1, 1)\}$, $\mathcal{K} = \mathbf{Z}_2 \times \{0\} = \{(1, 0), (0, 0)\}$ and $\mathcal{H} = \{0\} \times \mathbf{Z}_2 = \{(0, 0), (0, 1)\}$, where $\mathbf{Z}_2$ is a zero symmetric near ring under the binary operations addition mod(2) and multiplication mod(2). Also, here $\mathcal{K}$ and $\mathcal{H}$ are ideals of $\mathcal{N}$.
Consider the IFS A on $\mathcal{N}$ defined as
$$\mu_A((x, y)) = \begin{cases} 1 & \text{if } (x, y) \in \mathcal{K} \\ 0.5 & \text{if } (x, y) \notin \mathcal{K}, \end{cases} \qquad \nu_A((x, y)) = \begin{cases} 0 & \text{if } (x, y) \in \mathcal{K} \\ 0.3 & \text{if } (x, y) \notin \mathcal{K}. \end{cases}$$
Now, $\mathcal{G} = Aut(\mathcal{N}) = \{g_1 = i, g_2, g_3, g_4, g_5, g_6\}$ be the group of automorphisms of $\mathcal{N}$, where
$$g_1 = \{(0, 0) \to (0, 0); (0, 1) \to (0, 1); (1, 0) \to (1, 0); (1, 1) \to (1, 1)\}$$
$$g_2 = \{(0, 0) \to (0, 0); (0, 1) \to (1, 1); (1, 0) \to (1, 0); (1, 1) \to (0, 1)\}$$
$$g_3 = \{(0, 0) \to (0, 0); (0, 1) \to (0, 1); (1, 0) \to (1, 1); (1, 1) \to (1, 0)\}$$
$$g_4 = \{(0, 0) \to (0, 0); (0, 1) \to (1, 0); (1, 0) \to (0, 1); (1, 1) \to (1, 1)\}$$
$$g_5 = \{(0, 0) \to (0, 0); (0, 1) \to (1, 0); (1, 0) \to (1, 1); (1, 1) \to (0, 1)\}$$
$$g_6 = \{(0, 0) \to (0, 0); (0, 1) \to (1, 1); (1, 0) \to (0, 1); (1, 1) \to (1, 0)\}.$$

Now, it is easy to see that, for $g = g_1$ or $g_2 \in \mathcal{G}$, $A^g = A$ and for $g = g_4$ or $g_5 \in \mathcal{G}$, A^g is given by

$$\mu_{A^g}((x,y)) = \begin{cases} 1 & \text{if } (x,y) \in \mathcal{H} \\ 0.5 & \text{if } (x,y) \notin \mathcal{H}, \end{cases} \quad \nu_{A^g}((x,y)) = \begin{cases} 0 & \text{if } (x,y) \in \mathcal{H} \\ 0.3 & \text{if } (x,y) \notin \mathcal{H}. \end{cases}$$

Also, for $g = g_3$ or $g_6 \in \mathcal{G}$, A^g is given by

$$\mu_{A^g}((x,y)) = \begin{cases} 1 & \text{if } (x,y) \in \{(0,0),(1,1)\} \\ 0.5 & \text{if } (x,y) \notin \{(0,0),(1,1)\}, \end{cases} \quad \nu_{A^g}((x,y)) = \begin{cases} 0 & \text{if } (x,y) \in \{(0,0),(1,1)\} \\ 0.3 & \text{if } (x,y) \notin \{(0,0),(1,1)\}. \end{cases}$$

From the definition of group action on IFS, following results are easy to derive

Lemma 3.5. *Let A and B be two IFSs of a near ring $\mathcal{N}$ and $\mathcal{G}$ be a finite group which acts on A and B. Then for all $g, h \in \mathcal{G}$,*
(1) $(A \cap B)^g = A^g \cap B^g$, (2) $(A \cup B)^g = A^g \cup B^g$,
(3) $(A \times B)^g = A^g \times B^g$, (4) $A \subseteq B$ *implies* $A^g \subseteq B^g$,
(5) $(A^g)^h = A^{gh}$, (5) $(A^g)^{g^{-1}} = A^e$.

Lemma 3.6. *Let $\mathcal{G}$ be a finite group which acts on $\mathcal{N}$. Then for $x, y \in \mathcal{N}, g \in \mathcal{G}$, we have*
(1) $(x-y)^g = x^g - y^g$, (2) $(xy)^g = x^g y^g$, (3) $(x,y)^g = (x^g, y^g)$.

Proposition 3.7. *Let A be an IFI of a near ring $\mathcal{N}$ and $\mathcal{G}$ be a finite group which acts on A. Then A^g is also an IFI of $\mathcal{N}$.*

Proof. Let $x, y \in \mathcal{N}, g \in \mathcal{G}$ be any elements. Then

$$\begin{aligned} \mu_{A^g}(x-y) &= \mu_A((x-y)^g) = \mu_A(x^g - y^g) \\ &\geq min\{\mu_A(x^g), \mu_A(y^g)\} \\ &= min\{\mu_{A^g}(x), \mu_{A^g}(y)\}. \end{aligned}$$

Likewise, it can be revealed that $\nu_{A^g}(x-y) \leq max\{\nu_{A^g}(x), \nu_{A^g}(y)\}$. Also, we have

$$\begin{aligned} \mu_{A^g}(xy) &= \mu_A((xy)^g) = \mu_A(x^g y^g) \\ &\geq max\{\mu_A(x^g), \mu_A(y^g)\} \\ &= max\{\mu_{A^g}(x), \mu_{A^g}(y)\}. \end{aligned}$$

Likewise, it can be revealed that $\nu_{A^g}(xy) \leq min\{\nu_{A^g}(x), \nu_{A^g}(y)\}$. Thus A^g is an IF subnear ring of $\mathcal{N}$. In a same way, we can show that

$$\mu_{A^g}(x+y) \geq min\{\mu_{A^g}(x), \mu_{A^g}(y)\} \text{ and } \nu_{A^g}(x+y) \leq max\{\nu_{A^g}(x), \nu_{A^g}(y)\}.$$

[As $\mu_{A^g}(y) = \mu_A(y^g) = \mu_A(-y^g) = \mu_{A^g}(-y)$ and $\nu_{A^g}(y) = \nu_A(y^g) = \nu_A(-y^g) = \nu_{A^g}(-y)$].
So we have $\mu_{A^g}(x) = \mu_A(x^g) = \mu_A(y^g + x^g - y^g) = \mu_A((y+x-y)^g)$. Likewise, it can be revealed that $\nu_{A^g}(x) = \nu_A((y+x-y)^g)$. Hence A^g is an IFLI of $\mathcal{N}$.
For $x, y, z \in \mathcal{N}$, we have $\mu_{A^g}((x+z)-xy) = \mu_A((x^g+z^g)y^g - x^g y^g) \geq \mu_A(z^g) = \mu_{A^g}(z)$. Likewise, it can be revealed that $\nu_{A^g}((x+z)-xy) \leq \nu_{A^g}(z)$. This implies that A^g is an IFRI of $\mathcal{N}$. Therefore A^g is an IFI of $\mathcal{N}$. □

Remark 3.8. It is easy to check that the IFS A as defined in Example 3.4 is an IFI of $\mathcal{N}$. Also, for $g = g_1$ or g_2 or g_4 or $g_5 \in \mathcal{G}$, A^g is also an IFI of $\mathcal{N}$. However, for $g = g_3$ or $g_6 \in \mathcal{G}$, A^g is not an IFI of $\mathcal{N}$, for
$\mu_{A^{g_6}}((1,1)(0,1)) = \mu_{A^{g_6}}((0,1)) = 0.5 \ngeq 1 = \max\{\mu_{A^{g_6}}((1,1)), \mu_{A^6}((0,1))\} = \{1, 0.5\}$,
$\nu_{A^{g_6}}((1,1)(0,1)) = \nu_{A^{g_6}}((0,1)) = 0.3 \nleq 0 = \min\{\nu_{A^{g_6}}((1,1)), \nu_{A^{g_6}}((0,1))\} = \{0, 0.3\}$.

Definition 3.9. Let A be an IFI of a near ring $\mathcal{N}$ and $\mathcal{G}$ be a finite group which acts on A. Then A is called an *intuitionistic fuzzy $\mathcal{G}$-ideal* of $\mathcal{N}$, if A^g is an IFI of $\mathcal{N}$ for all $g \in \mathcal{G}$.

Example 3.10. Consider $\mathcal{N} = \mathbf{Z_6} = \{0,1,2,3,4,5\}$. Then $\mathcal{N}$ is a near-ring under the binary operations addition mod(6) and mulication mod(6). Take $\mathcal{K} = \{0,2,4\}$ be an ideal of $\mathcal{N}$. Define an IFS A on $\mathcal{N}$ as

$$\mu_A(x) = \begin{cases} 1 & \text{if } x \in \mathcal{K} \\ 0.5 & \text{if } x \notin \mathcal{K}, \end{cases} \quad \nu_A((x,y)) = \begin{cases} 0 & \text{if } x \in \mathcal{K} \\ 0.3 & \text{if } x \notin \mathcal{K}. \end{cases}$$

It is easy to check that A is an IFI of $\mathcal{N}$. Let $\mathcal{G} = Aut(\mathcal{N}) = \{i, g\}$ be the automorphism group of $\mathcal{N}$, where $g = \{1 \to 5, 2 \to 4, 3 \to 3, 4 \to 2, 5 \to 1, 0 \to 0\}$. Now it is easy to check that $A^g = A, \forall g \in \mathcal{G}$. So, A is an intuitionistic fuzzy $\mathcal{G}$-ideal of $\mathcal{N}$.

Proposition 3.11. *If A, B are two IFIs of a near ring $\mathcal{N}$ and $\mathcal{G}$ be a finite group which acts on A and B, then $(A \cap B)^g$ is also an IFI of $\mathcal{N}$.*

Proof. Follows from Lemma 3.5(1) and Proposition 3.7. □

Proposition 3.12. *If A, B are two IFIs of a near ring $\mathcal{N}$ and $\mathcal{G}$ be a finite group which acts on A and B, then $(A \times B)^g$ is also an IFI of $\mathcal{N}$.*

Proof. Follows from Lemma 3.5(3) and Proposition 3.7. □

Proposition 3.13. *If P is an intuitionistic fuzzy prime ideal of a near ring $\mathcal{N}$, then P^g is also an intuitionistic fuzzy prime ideal of $\mathcal{N}$, where $g \in \mathcal{G}$ be any element.*

Proof. Let A, B be IFIs of near ring $\mathcal{N}$ with $A \circ B \subseteq P^g$, where $g \in \mathcal{G}$ be any element. Now, we claim that $A^{g^{-1}} \circ B^{g^{-1}} \subseteq P$. It is sufficient to show that

$$\mu_{A^{g^{-1}} \circ B^{g^{-1}}}(x) \leq \mu_P(x) \text{ and } \nu_{A^{g^{-1}} B^{g^{-1}}}(x) \geq \nu_P(x), \ \forall x \in \mathcal{N}.$$

Now,

$$\begin{aligned} \mu_{A^{g^{-1}} \circ B^{g^{-1}}}(x) &= \sup_{x=ab} \{\min(\mu_{A^{g^{-1}}}(a), \mu_{B^{g^{-1}}}(b))\} \\ &= \sup_{x^{g^{-1}} = a^{g^{-1}} b^{g^{-1}}} \{\min(\mu_A(a^{g^{-1}}), \mu_B(b^{g^{-1}}))\} \\ &= \mu_{A \circ B}(x^{g^{-1}}) \\ &\leq \mu_{P^g}(x^{g^{-1}}) \\ &= \mu_P(x). \end{aligned}$$

Then $\mu_{A^{g^{-1}} \circ B^{g^{-1}}}(x) \leq \mu_P(x)$. Likewise, it can be revealed that $\nu_{A^{g^{-1}} \circ B^{g^{-1}}}(x) \geq \nu_P(x)$. Thus $A^{g^{-1}} \circ B^{g^{-1}} \subseteq P$ which implies that either $A^{g^{-1}} \subseteq P$ or $B^{g^{-1}} \subseteq P$, i.e., either $A \subseteq P^g$ or $B \subseteq P^g$, for if, $A^{g^{-1}} \subseteq P$, then $\mu_A(x) = \mu_A((x^g)^{g^{-1}}) = \mu_{A^{g^{-1}}}(x^g) \leq \mu_P(x^g) = \mu_{P^g}(x)$. Similarly, we have $\nu_A(x) \geq \nu_{P^g}(x)$. So $A \subseteq P^g$. Hence P^g is an IF prime ideal of $\mathcal{N}$. □

4. Intuitionistic fuzzy $\mathcal{G}$-prime ideal

By using the definition of $\mathcal{G}$-invariant ideal of a near ring $\mathcal{N}$, we define $\mathcal{G}$-invariant intuitionistic fuzzy ideal and $\mathcal{G}$-invariant intuitionistic fuzzy prime ideal of $\mathcal{N}$.

Definition 4.1. Let A be an IFS of a near ring $\mathcal{N}$ and $\mathcal{G}$ be a group which acts on $\mathcal{N}$. Then A is said to be *$\mathcal{G}$-invariant IFS* of $\mathcal{N}$, if

$$\mu_{A^g}(x) = \mu_A(x^g) \geq \mu_A(x), \nu_{A^g}(x) = \nu_A(x^g) \leq \nu_A(x) \ \forall x \in \mathcal{N} \text{ and } \forall g \in \mathcal{G}.$$

Proposition 4.2. *Let A be an IFS of a near ring $\mathcal{N}$ and $\mathcal{G}$ be a finite group which acts on $\mathcal{N}$. Then A is $\mathcal{G}$-invariant IFS of $\mathcal{N}$ if and only if $A^g = A$ for all $g \in \mathcal{G}$.*

Proof. Suppose A is $\mathcal{G}$-invariant IFS of $\mathcal{N}$ and let $x \in \mathcal{N}, g \in \mathcal{G}$. Then we have

$$\mu_A(x) = \mu_A((x^g)^{g^{-1}}) \geq \mu_A(x^g) \geq \mu_A(x).$$

Thus $\mu_A(x) = \mu_A(x^g) = \mu_{A^g}(x)$. Similarly, we have $\nu_A(x) = \nu_{A^g}(x)$. So $A^g = A$. The proof of the converse is easy. □

Theorem 4.3. *Let A be an IFS of a near ring $\mathcal{N}$ and $\mathcal{G}$ be a finite group which acts on $\mathcal{N}$. Let $A^{\mathcal{G}} = \bigcap_{g\in\mathcal{G}} A^g$. Then $A^{\mathcal{G}} = (\mu_{A^{\mathcal{G}}}, \nu_{A^{\mathcal{G}}})$, where $\mu_{A^{\mathcal{G}}}(x) = \min\{\mu_A(x^g) : g \in \mathcal{G}\}$ and $\nu_{A^{\mathcal{G}}}(x) = \max\{\nu_A(x^g) : g \in \mathcal{G}\}$ $\forall x \in \mathcal{N}$. Moreover, $A^{\mathcal{G}}$ is the largest $\mathcal{G}$-invariant IFS of $\mathcal{N}$ contained in A.*

Proof. Since A be an IFS of $\mathcal{N}$, A^g is also an IFS of $\mathcal{N}$ for all $g \in \mathcal{G}$. Also, intersection of IFSs of $\mathcal{N}$ is an IFS of $\mathcal{N}$ and then $A^{\mathcal{G}}$ is an IFS of $\mathcal{N}$. Next, we show that $A^{\mathcal{G}}$ is G-invariant IFS of $\mathcal{N}$. Now,

$$\begin{aligned}
\mu_{A^{\mathcal{G}}}(x^g) &= \min\{\mu_{A^h}(x^g) : h \in \mathcal{G}\} \\
&= \min\{\mu_A\{(x^g)^h\} : h \in \mathcal{G}\} \\
&= \min\{\mu_A(x^{gh}) : h \in \mathcal{G}\} \\
&= \min\{\mu_A(x^{g'}) : g' \in \mathcal{G}\} \\
&= \mu_{A^{\mathcal{G}}}(x).
\end{aligned}$$

Likewise, it can be revealed that $\nu_{A^{\mathcal{G}}}(x^g) = \nu_{A^{\mathcal{G}}}(x)$ $\forall x \in \mathcal{N}$. Thus $A^{\mathcal{G}}$ is $\mathcal{G}$-invariant IFS of $\mathcal{N}$.

Further, let B be any $\mathcal{G}$-invariant IFS of $\mathcal{N}$ such that $B \subseteq A$. Then for any $x \in \mathcal{N}, g \in \mathcal{G}$, we get $\mu_B(x^g) = \mu_B(x) \leq \mu_A(x)$ and $\nu_B(x^g) = \nu_B(x) \geq \nu_A(x)$.
Now, $\mu_B(x^g) = \mu_B(x) = \mu_B\{(x^g)^{g^{-1}}\} \leq \mu_A(x^g) \Rightarrow \mu_B(x) \leq \min\{\mu_A(x^g) : g \in \mathcal{G}\} = \mu_{A^{\mathcal{G}}}(x)$.
Similarly, $\nu_B(x^g) = \nu_B(x) = \nu_B\{(x^g)^{g^{-1}}\} \geq \nu_A(x^g) \Rightarrow \nu_B(x) \geq \max\{\nu_A(x^g) : g \in G\} = \nu_{A^{\mathcal{G}}}(x)$.
Thus $B \subseteq A^{\mathcal{G}}$. So $A^{\mathcal{G}}$ is the largest $\mathcal{G}$-invariant IFS of $\mathcal{N}$ contained in A. □

Proposition 4.4. *Let A be an IFI of a near ring $\mathcal{N}$ and $\mathcal{G}$ be a finite group which acts on $\mathcal{N}$. Then $A^{\mathcal{G}}$ is the largest $\mathcal{G}$-invariant IFI of $\mathcal{N}$ contained in A.*

Proof. Let $x, y \in \mathcal{N}, g \in \mathcal{G}$ be any element. Then

$$\begin{aligned}
\mu_{A^{\mathcal{G}}}(x-y) &= \min\{\mu_A((x-y)^g) : g \in \mathcal{G}\} \\
&= \min\{\mu_A(x^g - y^g) : g \in \mathcal{G}\} \\
&\geq \min\{\min\{\mu_A(x^g), \mu_A(y^g)\} : g \in \mathcal{G}\} \\
&= \min\{\min\{\mu_A(x^g) : g \in \mathcal{G}\}, min\{\mu_A(y^g) : g \in \mathcal{G}\}\} \\
&= \min\{\mu_{A^{\mathcal{G}}}(x), \mu_{A^{\mathcal{G}}}(y)\}.
\end{aligned}$$

Likewise, it can be revealed that $\nu_{A^{\mathcal{G}}}(x-y) \leq max\{\nu_{A^{\mathcal{G}}}(x), \nu_{A^{\mathcal{G}}}(y)\}$. Also,

$$\begin{aligned}
\mu_{A^{\mathcal{G}}}(xy) &= \min\{\mu_A((xy)^g) : g \in \mathcal{G}\} \\
&= \min\{\mu_A(x^g y^g) : g \in \mathcal{G}\} \\
&\geq \min\{max\{\mu_A(x^g), \mu_A(y^g)\} : g \in \mathcal{G}\} \\
&= \max\{\min\{\mu_A(x^g), \mu_A(y^g)\} : g \in \mathcal{G}\} \\
&= \max\{\min\{\mu_A(x^g) : g \in \mathcal{G}\}, min\{\mu_A(y^g) : g \in \mathcal{G}\}\} \\
&= \max\{\mu_{A^{\mathcal{G}}}(x), \mu_{A^{\mathcal{G}}}(y)\}.
\end{aligned}$$

Likewise, it can be revealed that $\nu_{A^{\mathcal{G}}}(xy) \leq min\{\nu_{A^{\mathcal{G}}}(x), \nu_{A^{\mathcal{G}}}(y)\}$. Further

$$\begin{aligned}
\mu_{A^{\mathcal{G}}}(y+x-y) &= \min\{\mu_A((y+x-y)^g) : g \in \mathcal{G}\} \\
&= \min\{\mu_A(y^g + x^g - y^g) : g \in \mathcal{G}\} \\
&= \min\{\mu_A(x^g) : g \in \mathcal{G}\} \\
&= \mu_{A^{\mathcal{G}}}(x).
\end{aligned}$$

Likewise, it can be revealed that $\nu_{A^{\mathcal{G}}}(y+x-y)=\nu_{A^{\mathcal{G}}}(x)$.

$$\begin{aligned}\mu_{A^{\mathcal{G}}}((x+z)y-xy) &= \min\{\mu_A(((x+z)y-xy)^g) : g\in\mathcal{G}\}\\ &= \min\{\mu_A((x+z)^g y^g - x^g y^g) : g\in\mathcal{G}\}\\ &\geq \min\{\mu_A((x^g+z^g)y^g - x^g y^g) : g\in\mathcal{G}\}\\ &= \min\{\mu_A(z^g) : g\in\mathcal{G}\}\\ &= \mu_{A^{\mathcal{G}}}(z).\end{aligned}$$

Likewise, it can be revealed that $\nu_{A^{\mathcal{G}}}((x+z)y-xy)\leq\nu_{A^{\mathcal{G}}}(z)$. Thus $A^{\mathcal{G}}$ is an IFI of $\mathcal{N}$. Further, $A^{\mathcal{G}}$ is the largest $\mathcal{G}$-invariant IFI of $\mathcal{N}$ contained in A can be proved similar to Theorem 4.3. □

Proposition 4.5. *An IFI A of a near ring $\mathcal{N}$ is $\mathcal{G}$-invariant IFI of $\mathcal{N}$ if and only if $A^{\mathcal{G}}=A$.*

Proof. Suppose A is $\mathcal{G}$-invariant IFI of $\mathcal{N}$. Then from Proposition 4.4, we get $A^{\mathcal{G}}\subseteq A$. Since $A\subseteq A$, by the hypothesis, $A\subseteq A^{\mathcal{G}}$. Thus $A^{\mathcal{G}}=A$. The proof of the converse is easy. □

Example 4.6. Consider $\mathcal{N}$, $\mathcal{G}$ and the intuitionistic fuzzy set A same as in Example 3.10. Then it is easy to check that $A^{\mathcal{G}}=A$. Thus A is an $\mathcal{G}$-invariant IFI of $\mathcal{N}$.

Theorem 4.7. *If A,B are G-invariant IFIs of a near ring $\mathcal{N}$, then $A+B$ is also $\mathcal{G}$-invariant IFI of $\mathcal{N}$.*

Proof. Let $x\in\mathcal{N}, g\in\mathcal{G}$. Then we have

$$\begin{aligned}\mu_{(A+B)^g}(x) &= \mu_{A+B}(x^g)\\ &= \sup_{x^g=a+b}\{\mu_A(a),\mu_B(b)\}\\ &= \sup_{x^g=a+b}\{\mu_A(a^{g^{-1}}),\mu_B(b^{g^{-1}})\}\\ &= \sup_{x=a^{g^{-1}}+b^{g^{-1}}}\{\mu_A(a^{g^{-1}}),\mu_B(b^{g^{-1}})\}\\ &= \mu_{A+B}(x).\end{aligned}$$

Likewise, it can be revealed that $\nu_{(A+B)^g}(x)=\nu_{A+B}(x)$. Thus $(A+B)^g=A+B\ \forall g\in\mathcal{G}$. So $A+B$ is also $\mathcal{G}$-invariant IFI of $\mathcal{N}$. □

Theorem 4.8. *If A,B are $\mathcal{G}$-invariant IFIs of a near ring $\mathcal{N}$, then $A\circ B$ is also $\mathcal{G}$-invariant IFI of $\mathcal{N}$.*

Proof. Let $x\in\mathcal{N}, g\in\mathcal{G}$. Then we get

$$\begin{aligned}\mu_{(A\circ B)^g}(x) &= \sup_{x^g=ab}\min\{\mu_A(a),\mu_B(b)\}\\ &= \sup_{x^g=ab}\min\{\mu_A(a^{g^{-1}}),\mu_B(b^{g^{-1}})\}\\ &= \sup_{x=a^{g^{-1}}b^{g^{-1}}}\min\{\mu_A(a^{g^{-1}}),\mu_B(b^{g^{-1}})\}\\ &= \mu_{A\circ B}(x).\end{aligned}$$

Likewise, it can be revealed that $\nu_{(A\circ B)^g}(x)=\nu_{A\circ B}(x)$. Thus $(A\circ B)^g=A\circ B\ \forall g\in\mathcal{G}$. So $A\circ B$ is also $\mathcal{G}$-invariant IFI of $\mathcal{N}$. □

Definition 4.9. Let P be a non-constant IFI of a near ring $\mathcal{N}$ and $\mathcal{G}$ be a finite group which acts on P. Then P is termed as an *IF $\mathcal{G}$-prime ideal* of $\mathcal{N}$, if P is $\mathcal{G}$-invariant IF prime ideal of $\mathcal{N}$.

Proposition 4.10. *Let P be an IF $\mathcal{G}$-prime ideal of $\mathcal{N}$. Then $P_{(s,t)}$ is a $\mathcal{G}$-prime ideal of $\mathcal{N}$, where $s\in[\mu_P(1),\mu_P(0)]$ and $t\in[\nu_P(0),\nu_P(1)]$ such that $s+t\leq 1$.*

Proof. It is easy to show that $P_{(s,t)}$ is an ideal of $\mathcal{N}$. We show that $P_{(s,t)}$ is $\mathcal{G}$-invariant.
Let $x \in P_{(s,t)}, g \in \mathcal{G}$ be any element. Since P is $\mathcal{G}$-invariant intuitionistic fuzzy prime ideal of $\mathcal{N}$, $\mu_P(x^g) = \mu_P(x) \geq s$ and $\nu_P(x^g) = \nu_P(x) \leq t \ \forall g \in \mathcal{G}$. Then $x^g \in P_{(s,t)}, \forall g \in \mathcal{G}$. Thus $P_{(s,t)}$ is $\mathcal{G}$-invariant.

Next we show that $P_{(s,t)}$ is prime ideal of $\mathcal{N}$. Let I and J be two $\mathcal{G}$-invariant ideals of $\mathcal{N}$ such that $IJ \subseteq P_{(s,t)}$. Define two IFSs $A = \chi_I$ and $B = \chi_J$. It is easy to check that A and B are $\mathcal{G}$-invariant IFIs of $\mathcal{N}$ (as I and J are $\mathcal{G}$-invariant ideals). We claim that $A \circ B \subseteq P$. Let $x \in \mathcal{N}$ be any element. If $A \circ B(x) = (0,1)$, then there is nothing to prove. If $A \circ B(x) \neq (0,1)$, then we have

$$\mu_{A\circ B}(x) = \sup_{x=yz} \min\{\mu_A(y), \mu_B(z)\} = \sup_{x=yz} \min\{\mu_{\chi_I}(y), \mu_{\chi_J}(z)\} \neq 0$$

and

$$\nu_{A\circ B}(x) = \inf_{x=yz} \max\{\nu_A(y), \nu_B(z)\} = \inf_{x=yz} \max\{\nu_{\chi_I}(y), \nu_{\chi_J}(z)\} \neq 1.$$

This implies that there exist $y \in I, z \in J$ such that $x = yz$. Moreover, $A \circ B(x) = (s,t)$. Thus $x = yz \in IJ \subseteq P_{(s,t)}$. So $\mu_P(x) \geq s, \nu_P(x) \leq t$. Hence $A \circ B \subseteq P$. Since P is an intuitionistic fuzzy $\mathcal{G}$-prime ideal of $\mathcal{N}$, either $A \subseteq P$ or $B \subseteq P$. Suppose $A \subseteq P$. Then $I \subseteq P_{(s,t)}$. For, assume that $I \supset P_{(s,t)}$. Then there is an element $a \in \mathcal{N}$ such that $a \in I$, but $a \notin P_{(s,t)}$. This implies that $\mu_A(a) = \mu_{\chi_I}(a) = s$ and $\nu_A(a) = \nu_{\chi_I}(a) = t$, but $\mu_P(a) < s$ and $\nu_P(a) > t$. Thus $\mu_A(a) = s > \mu_P(a)$ and $\nu_A(a) = t < \nu_P(a)$. So $A \supset P$, a contradiction. Similarly, we have $B \subseteq P$. Hence $P_{(s,t)}$ is $\mathcal{G}$-prime ideal of $\mathcal{N}$. □

From the above discussion on the results on intuitionistic fuzzy prime ideals that are $\mathcal{G}$-invariant also. We can also define intuitionistic fuzzy $\mathcal{G}$-prime ideals in the following ways too

Definition 4.11. A non-constant $\mathcal{G}$-invariant IFI P of a near ring $\mathcal{N}$ is said to be a *$\mathcal{G}$-prime IFI*, if for any two $\mathcal{G}$-invariant IFIs A and B of $\mathcal{N}$ such that $A \circ B \subseteq P$, either $A \subseteq P$ or $B \subseteq P$.

Proposition 4.12. *Let P be an intuitionistic fuzzy $\mathcal{G}$-invariant ideal of near ring $\mathcal{N}$. Then the following are equivalent:*

(1) *P is an intuitionistic fuzzy $\mathcal{G}$-prime ideal of $\mathcal{N}$,*
(2) *For any $x_{(p,q)}, y_{(s,t)} \in IFP(\mathcal{N})$, where $x, y \in \mathcal{N}$ are $\mathcal{G}$-invariant points such that $x_{(p,q)}y_{(s,t)} \subseteq P$ implies that either $x_{(p,q)} \subseteq P$ or $y_{(s,t)} \subseteq P$.*

Proof. (1) $\Rightarrow$ (2) Suppose P is an intuitionistic fuzzy $\mathcal{G}$-prime ideal of $\mathcal{N}$. Let $x_{(p,q)}, y_{(s,t)} \in IFP(\mathcal{N})$, where $x, y \in \mathcal{N}$ are $\mathcal{G}$-invariant points such that $x_{(p,q)}y_{(s,t)} \subseteq P$. Then $x_{(p,q)}y_{(s,t)} = (xy)_{(p\wedge s, q\vee t)}$, where $\mu_P(xy) \geq p \wedge s$ and $\nu_P(xy) \leq q \vee t$.
Let us define IFSs A, B of $\mathcal{N}$ by $A = \chi_{<x>}$ and $B = \chi_{<y>}$. Clearly, A and B are $\mathcal{G}$-invariant IFIs of $\mathcal{N}$. Now we have

$$\mu_{A\circ B}(z) = \sup_{z=uv} \min\{\mu_A(u), \mu_B(v)\} = p \wedge s \text{ and } \nu_{AoB}(z) = \inf_{z=uv} \max\{\nu_A(u), \nu_B(v)\} = q \vee t,$$

where $u \in < x >$ and $v \in < y >$. Thus $\mu_{A\circ B}(z) = p \wedge s \leq \mu_P(z)$ and $\nu_{A\circ B}(z) = q \vee t \geq \nu_P(z)$, when $z = uv$, where $u \in < x >$ and $v \in < y >$. Otherwise $A \circ B(z) = (0,1)$, i.e., $A \circ B \subseteq P$. As P is intuitionistic fuzzy $\mathcal{G}$-prime ideal so either $A \subseteq P$ or $B \subseteq P$. So $x_{(p,q)} \subseteq A \subseteq P$ or $y_{(s,t)} \subseteq B$.

(2) $\Rightarrow$ (1) Let A and B be two $\mathcal{G}$-invariant IFIs of $\mathcal{N}$ such that $A \circ B \subseteq P$. Suppose $A \supset P$. Then there exists $\mathcal{G}$-invariant element $x \in \mathcal{N}$ such that $\mu_A(x) > \mu_P(x)$ and $\nu_A(x) < \nu_P(x)$. Let $\mu_A(x) = p, \nu_A(x) = q$. Let $y \in \mathcal{N}$ be $\mathcal{G}$-invariant element of $\mathcal{N}$ such that $\mu_A(x) = r, \nu_A(x) = s$. If $z = xy$, then $x_{(p,q)}y_{(s,t)} = (xy)_{(p\wedge s, q\vee t)}$. Thus we get

$$\mu_P(z) = \mu_P(xy) \geq \mu_{A\circ B}(xy) \geq \min\{\mu_A(x), \mu_B(y)\} = p \wedge s = \mu_{(xy)_{(p\wedge s, q\vee t)}}(z).$$

Similarly, we have $\nu_P(z) \leq \nu_{(xy)_{(p\wedge s,q\vee t)}}(z)$. So $x_{(p,q)}y_{(s,t)} \subseteq P$. By (2), we get either $x_{(p,q)} \subseteq P$ or $y_{(s,t)} \subseteq P$, i.e., either $\mu_P(x) \geq p$, $\nu_P(x) \leq q$ or $\mu_P(x) \geq s$, $\nu_P(x) \leq t$. Since $\mu_P(x) < p, \nu_P(x) > q$ and $\mu_B(y) = s \leq \mu_P(y)$, $\nu_B(y) = t \geq \nu_P(y)$. Hence $B \subseteq P$. Therefore P is an intuitionistic fuzzy $\mathcal{G}$-prime ideal of $\mathcal{N}$. □

Definition 4.13. Let $\{A_n : n \in I\}$ be a non-empty family of IFSs of a near ring $\mathcal{N}$. Then define the IFS $\bigcup_{n\in I} A_n = (\bigcup_{n\in I} \mu_{A_n}, \bigcup_{n\in I} \nu_{A_n})$ of $\mathcal{N}$ as follow:

$$\mu_{\bigcup_{n\in I} A_n}(x) = \bigvee_{n\in I} \mu_{A_n}(x) \text{ and } \nu_{\bigcup_{n\in I} A_n}(x) = \bigwedge_{n\in I} \nu_{A_n}(x) \text{ for each } x \in \mathcal{N}.$$

Example 4.14. Let $\mathcal{N} = \mathbb{Z}_{p^m}$. Then $(\mathcal{N}, +_{p^m}, \times_{p^m})$ form a near-ring.
For each $k \in \mathbb{N}$, let $I_k = \{x \in \mathbb{Z}_{p^k} | x \equiv 0(mod(p^m))\}$. Clearly, $I_1 \subseteq I_2 \subseteq I_3 \subseteq \cdots \subseteq I_k \subseteq I_{k+1} \subseteq \cdots$ form a chain of ideals of $\mathcal{N}$. Define an IFS $A_k = (\mu_{A_k}, \nu_{A_k})$ of $\mathcal{N}$ as follows:

$$\mu_{A_k}(x) = \begin{cases} \frac{gcd\{x,p^k\}}{p^k} & \text{if } p|x \\ 0 & \text{if } p \nmid x, \end{cases} \qquad \nu_{A_k}(x) = \begin{cases} 1 - \mu_{A_k}(x) & \text{if } p|x \\ 1 & \text{if } p \nmid x. \end{cases}$$

Then it is easy to see that $\{A_k : k = 1, 2, 3, \cdots, m\}$ forms a chain of IFIs of near ring $\mathcal{N}$.

Following the Theorem 2.7 in [19], it is easy to prove the following Lemma

Lemma 4.15. *Let $\{A_n : n \in I\}$ be a chain of IFIs of $\mathcal{N}$. Then $\bigcup_{n\in I} A_n$ is an IFI of $\mathcal{N}$.*

Proposition 4.16. *If P is an IF prime ideal of a near ring $\mathcal{N}$, then $P^{\mathcal{G}}$ is $\mathcal{G}$-prime IFI ideal of $\mathcal{N}$. Conversely, if Q is an intuitionistic fuzzy $\mathcal{G}$-prime ideal of $\mathcal{N}$, then there exists an IF prime ideal P of $\mathcal{N}$ such that $P^{\mathcal{G}} = Q$ where P is unique up to $\mathcal{G}$-orbits.*

Proof. Suppose P is an intuitionistic fuzzy prime ideal of $\mathcal{N}$ and let A and B be two $\mathcal{G}$-invariant IFIs of $\mathcal{N}$ such that $A \circ B \subseteq P^{\mathcal{G}}$. Then $A \circ B \subseteq P$ (since $P^{\mathcal{G}} \subseteq P$ always). Thus either $A \subseteq P$ or $B \subseteq P$. But $P^{\mathcal{G}}$ is the largest $\mathcal{G}$-invariant IFI of $\mathcal{N}$ contained in P. So either $A \subseteq P^{\mathcal{G}}$ or $B \subseteq P^{\mathcal{G}}$. Hence $P^{\mathcal{G}}$ is $\mathcal{G}$-prime IFI ideal of $\mathcal{N}$.

Conversely, suppose Q is an IF $\mathcal{G}$-prime ideal of $\mathcal{N}$. Then $Q^{\mathcal{G}} \subseteq Q$. Let $\mathcal{S} = \{P | P$ is an IFI of $\mathcal{N}$ with $P^{\mathcal{G}} \subseteq Q\}$. Clearly $\mathcal{S} \neq \varnothing$. Let $\mathcal{C} = \{A_i | i \in I\}$ be a chain of members of $\mathcal{S}$. Then by Lemma 4.15, $\bigcup_{i\in I} A_i$ is an IFI of $\mathcal{N}$. Also for any $x \in \mathcal{N}$, we have

$$\begin{aligned} \mu_{(\bigcup_{i\in I} A_i)^{\mathcal{G}}}(x) &= \min_{g\in\mathcal{G}}\{\mu_{\bigcup_{i\in I} A_i}(x^g)\} \\ &= \min_{g\in\mathcal{G}}\{\bigvee_{i\in I} \mu_{A_i}(x^g)\} \\ &= \bigvee_{i\in I} \min_{g\in\mathcal{G}}\{\mu_{A_i}(x^g)\} \\ &= \bigvee_{i\in I} \mu_{A_i^{\mathcal{G}}}(x)[\because A_i^G \subseteq Q] \\ &\leq \mu_Q(x). \end{aligned}$$

Similarly, we can show that $\nu_{(\bigcup_{i\in I} A_i)^{\mathcal{G}}}(x) \geq \nu_Q(x)$. Thus $(\bigcup_{i\in I} A_i)^{\mathcal{G}} \subseteq Q$. So $(\bigcup_{i\in I} A_i)^{\mathcal{G}} \in \mathcal{S}$.

Now, at this point we can apply Zorn's lemma to the set $\mathcal{S}$. Let there exist an intuitionistic fuzzy maximal ideal P such that $P^{\mathcal{G}} \subseteq Q$. Let A and B be two IFIs of $\mathcal{N}$ such that $A \circ B \subseteq P$. Then $(A \circ B)^{\mathcal{G}} \subseteq P^{\mathcal{G}} \subseteq Q$. Since $A^{\mathcal{G}}$ and $B^{\mathcal{G}}$ are largest IFIs of $\mathcal{N}$ contained in A and B respectively. We claim that $A^{\mathcal{G}} \circ B^{\mathcal{G}} \subseteq A \circ B$ is a $\mathcal{G}$-invariant.

$$\begin{aligned} \mu_{A^{\mathcal{G}} \circ B^{\mathcal{G}}}(x^g) &= \sup_{x^g=uv} \min\{\mu_{A^{\mathcal{G}}}(u), \mu_{B^{\mathcal{G}}}(v)\} \\ &= \sup_{x=u^{g^{-1}}v^{g^{-1}}} \min\{\mu_{A^{\mathcal{G}}}(u^{g^{-1}}), \mu_{B^{\mathcal{G}}}(v^{g^{-1}})\} \\ &= \mu_{A^{\mathcal{G}} \circ B^{\mathcal{G}}}(x). \end{aligned}$$

Similarly, we can show that $\nu_{A^{\mathcal{G}} \circ B^{\mathcal{G}}}(x^g) = \nu_{A^{\mathcal{G}} \circ B^{\mathcal{G}}}(x)$. Thus $A^{\mathcal{G}} \circ B^{\mathcal{G}} \subseteq (A \circ B)^G \subseteq Q$. Since Q is an IF $\mathcal{G}$ prime ideal of $\mathcal{N}$, we have either $A^{\mathcal{G}} \subseteq Q$ or $B^{\mathcal{G}} \subseteq Q$. By maximality of P, either $A \subseteq P$ or $B \subseteq P$. This implies that P is an IF prime ideal of $\mathcal{N}$. As $Q^{\mathcal{G}} \subseteq Q$, we have $Q \in \mathcal{S}$. But maximality of P gives that $Q \subseteq P$. Since P and $Q^{\mathcal{G}}$ are $\mathcal{G}$ invariant and $P^{\mathcal{G}}$ is largest in P, we get $Q \subseteq P^{\mathcal{G}}$. So $P^{\mathcal{G}} = Q$.

Suppose there exists another IF prime ideal T of $\mathcal{N}$ such that $T^{\mathcal{G}} = Q$. Since $\circ_{g\in\mathcal{G}} P^g \subseteq \bigcap_{g\in\mathcal{G}} P^g = P^{\mathcal{G}} \subseteq T$, $P^g \subseteq T$ for some $g \in \mathcal{G}$ by the primeness of T. Then $P \subseteq T^{g^{-1}}$ and thus

$P = T^{g^{-1}}$. Since the fact that $(T^{g^{-1}})^{\mathcal{G}} = T^{\mathcal{G}} \subseteq Q$ implies that T is contained in the set $\mathcal{S}$. So P is unique up to G-orbits. □

5. $\mathcal{G}$-Homomorphism of intuitionistic fuzzy $\mathcal{G}$-ideals

In this part of the paper, we explore the image and pre image of intuitionistic fuzzy $\mathcal{G}$-ideals under the near ring homomorphism.

Definition 5.1. A near homomorphism $\phi : \mathcal{N} \to \mathcal{N}'$ from a ring $\mathcal{N}$ to a ring $\mathcal{N}'$ with unity is called a *$\mathcal{G}$-homomorphism*, if for all $g \in \mathcal{G}, x \in \mathcal{N}, \phi(g * x) = g * \phi(x)$, where group $\mathcal{G}$ acts on both the near rings.

Lemma 5.2. *Let $\mathcal{N}$ and $\mathcal{N}'$ be near rings and $\mathcal{G}$ be a finite group which acts on $\mathcal{N}$ and $\mathcal{N}'$. Let $f : \mathcal{N} \to \mathcal{N}'$ is a function defined by $f(x^g) = (f(x))^g\ \forall x \in \mathcal{N}, g \in \mathcal{G}$. Then f is a near ring homomorphism. Moreover, f is also a $\mathcal{G}$-homomorphism.*

Proof. Let $x, y \in \mathcal{N}, g \in \mathcal{G}$ be any elements. Then we have

$$f(x^g + y^g) = f((x+y)^g) = (f(x)+f(y))^g = (f(x))^g + (f(y))^g = f(x^g) + f(y^g) \text{ and}$$
$$f(x^g y^g) = f((xy)^g) = (f(x)f(y))^g = (f(x))^g (f(y))^g = f(x^g)f(y^g).$$

Thus f is a near ring homomorphism. Also, $f(g * x) = f(x^g) = (f(x))^g = g * f(x)$ revealed that f is also a $\mathcal{G}$-homomorphism. □

Lemma 5.3. *Let $\mathcal{N}$ and $\mathcal{N}'$ be near rings and $\mathcal{G}$ be a finite group which acts on $\mathcal{N}$ and $\mathcal{N}'$. Let $f : \mathcal{N} \to \mathcal{N}'$ be a $\mathcal{G}$-homomorphism and A, B are IFSs of $\mathcal{N}$ and $\mathcal{N}'$ respectively. Then*
- (1) *$f^{-1}(B^g) = (f^{-1}(B))^g\ \forall g \in \mathcal{G}$,*
- (2) *$f(A^g) = (f(A))^g\ \forall g \in \mathcal{G}$.*

Proof. (1) Let $x \in \mathcal{N}$ and $g \in \mathcal{G}$. Then

$$f^{-1}(B^g)(x) = B^g(f(x)) = B((f(x))^g) = B(f(x^g))) = f^{-1}(B)(x^g) = (f^{-1}(B))^g(x)$$

Thus $f^{-1}(B^g) = (f^{-1}(B))^g\ \forall g \in \mathcal{G}$.

(2) Let $y \in \mathcal{N}'$ and $g \in \mathcal{G}$t. Then $f(A^g)(y) = (\mu_{f(A^g)}(y), \nu_{f(A^g)}(y))$. Now

$$\begin{aligned}
\mu_{f(A^g)}(y) &= \sup\{\mu_{A^g}(x) : f(x) = y\} = \sup\{\mu_A(x^g) : f(x) = y\} \\
&= \sup\{\mu_A(x^g) : f(x^g) = y^g\} \\
&= \mu_{f(A)}(f(x^g)) = \mu_{f(A)}((f(x))^g) = \mu_{(f(A))^g}(f(x)) = \mu_{(f(A))^g}(y).
\end{aligned}$$

Similarly, we can show that $\nu_{f(A^g)}(y) = \mu_{(f(A))^g}(y)$. Thus $f(A^g) = (f(A))^g\ \forall g \in \mathcal{G}$. □

By using Lemma 5.3, one can easily derive the following two results:

Theorem 5.4. *Let $\mathcal{N}$ and $\mathcal{N}'$ be near rings and $\mathcal{G}$ be a finite group which acts on $\mathcal{N}$ and $\mathcal{N}'$. Let $f : \mathcal{N} \to \mathcal{N}'$ be a $\mathcal{G}$-homomorphism. If B be an intuitionistic fuzzy $\mathcal{G}$-ideal of $\mathcal{N}'$, then $f^{-1}(B)$ is an intuitionistic fuzzy $\mathcal{G}$-ideal of $\mathcal{N}$.*

Theorem 5.5. *Let $\mathcal{N}$ and $\mathcal{N}'$ be near rings and $\mathcal{G}$ be a finite group which acts on $\mathcal{N}$ and $\mathcal{N}'$. Let $f : \mathcal{N} \to \mathcal{N}'$ be a $\mathcal{G}$-epimorphism. If A is an intuitionistic fuzzy $\mathcal{G}$-ideal of $\mathcal{N}$ which is constant on $Kerf$ of $\mathcal{N}$, then $f(A)$ is an intuitionistic fuzzy $\mathcal{G}$-ideal of $\mathcal{N}'$.*

Theorem 5.6. *Let $\mathcal{N}$ and $\mathcal{N}'$ be near rings and $\mathcal{G}$ be a finite group which acts on $\mathcal{N}$ and $\mathcal{N}'$. Let $f : \mathcal{N} \to \mathcal{N}'$ be a $\mathcal{G}$-homomorphism. If P be an intuitionistic fuzzy $\mathcal{G}$-prime ideal of $\mathcal{N}'$, then $f^{-1}(P)$ is an intuitionistic fuzzy $\mathcal{G}$-prime ideal of $\mathcal{N}$.*

Proof. Suppose P be an intuitionistic fuzzy $\mathcal{G}$-prime ideal of $\mathcal{N}'$. Then by Theorem 2.14, $f^{-1}(P)$ is also an intuitionistic fuzzy prime ideal of $\mathcal{N}$. It remain to show that $f^{-1}(P)$ is $\mathcal{G}$-invariant. Let $x \in \mathcal{N}, g \in \mathcal{G}$. Then we have $\mu_{f^{-1}(P)}(x^g) = \mu_P(f(x^g)) = \mu_P((f(x))^g) = \mu_P((f(x))) = \mu_{f^{-1}(P)}(x)$. Likewise, it can be revealed that $\nu_{f^{-1}(P)}(x^g) = \nu_{f^{-1}(P)}(x)$. Thus $f^{-1}(P)$ is $\mathcal{G}$-invariant. So $f^{-1}(P)$ is an intuitionistic fuzzy $\mathcal{G}$-prime ideal of $\mathcal{N}$. □

Theorem 5.7. *Let $\mathcal{N}$ and $\mathcal{N}^{'}$ be near rings and $\mathcal{G}$ be a finite group which acts on $\mathcal{N}$ and $\mathcal{N}^{'}$. Let $f : \mathcal{N} \to \mathcal{N}^{'}$ be a $\mathcal{G}$-epimorphism. If P is an intuitionistic fuzzy $\mathcal{G}$-prime ideal which is constant on $Kerf$ of $\mathcal{N}$, then $f(P)$ is an intuitionistic fuzzy $\mathcal{G}$-prime ideal of $\mathcal{N}^{'}$.*

Proof. Suppose P is an intuitionistic fuzzy $\mathcal{G}$-prime ideal of $\mathcal{N}$ which is constant on $Kerf$ of $\mathcal{N}$. Then by Theorem 2.15, $f(P)$ is also an intuitionistic fuzzy prime ideal of $\mathcal{N}^{'}$. It remain to show that $f(P)$ is $\mathcal{G}$-invariant. Let $y \in \mathcal{N}^{'}, g \in \mathcal{G}$. As f is an epimorphism, there exists $x \in \mathcal{N}$ such that $f(x) = y$. Thus we have
$\mu_{f(P)}(y^g) = \mu_{(f(P))^g}(y) = \mu_{f(P^g)}(y) = \mu_{P^g}(f^{-1}(y)) = \mu_{P^g}(x) = \mu_P(x^g) = \mu_P(x) = \mu_P(f^{-1}(y)) = \mu_{f(P)}(y)$. Similarly, we can show that $\nu_{f(P)}(y^g) = \nu_{f(P)}(y)$. So $f(P)$ is $\mathcal{G}$-invariant. Hence $f(P)$ is an intuitionistic fuzzy $\mathcal{G}$-prime ideal of $\mathcal{N}^{'}$. □

6. Conclusions

In this article, we studied the effect of group action on the intuitionistic fuzzy ideals of a near ring and developed a relation between the intuitionistic fuzzy $\mathcal{G}$-prime ideals and the intuitionistic fuzzy prime ideals of $\mathcal{N}$. We also established that the largest $\mathcal{G}$-invariant intuitionistic fuzzy ideal contained in an intuitionistic fuzzy prime ideal is an intuitionistic fuzzy $\mathcal{G}$-prime ideal of $\mathcal{N}$. Conversely, if Q is an intuitionistic fuzzy $\mathcal{G}$-prime ideal of $\mathcal{N}$, then there exists an intuitionistic fuzzy prime ideal P of $\mathcal{N}$ such that $P^{\mathcal{G}} = Q$ where P is unique up to $\mathcal{G}$-orbits. We also investigated the relationship between the intuitionistic fuzzy $\mathcal{G}$-prime ideals of $\mathcal{N}$ and their level cut sets under this group action. Additionally, we developed a suitable characterization of intuitionistic fuzzy $\mathcal{G}$-prime ideals of $\mathcal{N}$ in terms of intuitionistic fuzzy points of $\mathcal{N}$ under this group action. In addition to these, we have also investigated the behavior of an intuitionistic fuzzy $\mathcal{G}$-prime ideal of a near ring under $\mathcal{G}$-homomorphism. In the future, we plan to study partial group action (the existence of $g * (h * x)$ implies the existence of $(gh) * x$, but not necessarily conversely) on intuitionistic fuzzy ideals of near rings. The Propositions that we prove are the following which are generalizations of Propositions 4.5 and 4.13.

Open Problem 1. Can we establish relation between $\mathcal{G}$-invariant intuitionistic fuzzy ideal and largest $\mathcal{G}$-invariant intuitionistic fuzzy ideal of $\mathcal{N}$ under partial group action?

Open Problem 2. Can we investigate relationship between primeness and $\mathcal{G}$-primeness of an intuitionistic fuzzy ideal if a group $\mathcal{G}$ partially acts on an intuitionistic fuzzy ideal?

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